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08:3009:30	Prof. Weinan E
09:3009:35	Break
09:3510:10	Prof. Chieh-Sen Huang
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10:5011:25	Prof. Matheus Tuwankotta
11:2512:00	Prof. Tao Zhou

Plenary Talk

A Mathematical Perspective of

Machine Learning



Prof. Weinan E Peking University

The heart of modern machine learning (ML) is the high dimensional functions. Traditional approximation of approaches, such as approximation by piecewise polynomials, wavelets, or other linear combinations of fixed basis functions, suffer from the curse of dimensionality (CoD). This does not seem to be the case for the neural network-based ML models. To quantify this, we need to develop the corresponding mathematical framework. In this talk, I will report the progress made so far and the main remaining issues within the scope of supervised learning.I will discuss three major issues: approximation theory and error analysis of modern ML models, qualitative behavior of gradient descent algorithms, and ML from a continuous viewpoint.

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& Self-Adaptive Theta Scheme using Discontinuity Aware Quadrature for

Solving Conservation Laws



Prof. Chieh-Sen Huang National Sun Yat-sen University

We present a \emph{discontinuity aware quadrature} (DAQ) rule, and use it to develop implicit \emph{self-adaptive theta} (SATh) schemes for the approximation of scalar hyperbolic conservation laws. Our SATh schemes require the solution of a system of two equations, one controlling the cell averages of the solution at the time levels, and the other controlling the space-time averages of the solution. These quantities are used within the DAQ rule to approximate the time integral of the hyperbolic flux function accurately, even when the solution may be discontinuous somewhere over the time interval. The result is a finite volume scheme using the theta time stepping method, with theta defined implicitly (or self-adaptively). Two schemes are developed, SATh-up for a monotone flux function using upstream stabilization, and SATh-LF using simple the Lax-Friedrichs numerical flux. We prove that DAQ is accurate to second order when there is a discontinuity in the solution and third order when it is smooth. We prove that SATh-up is unconditionally stable, provided that theta is set to be at least 1/2(which means that SATh can be only first order accurate in general). We also prove that SATh-up satisfies the maximum principle and is total variation diminishing under appropriate monotonicity and boundary conditions. General flux functions require the SATh-LF scheme, so we assess its accuracy through numerical examples in one and two space dimensions. These results suggest that SATh-LF is also stable and satisfies the maximum principle (at least at reasonable CFL numbers). Compared to solutions of finite volume schemes using Crank-Nicolson and backward Euler time stepping, SATh-LF solutions often approach

the accuracy of the former but without oscillation, and they are numerically less diffuse than the later.

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Mathematical medicine linking shape and skin disease



Prof. S. Seirin-Lee Kyoto University

Urticaria is a common skin disease characterized by the rapid appearance and disappearance of local skin edema and flares with itching. It affects about one in 5 people at some point in their lives and globally about 56/100000 population suffer from urticaria daily. It is characterized by various macroscopic skin eruptions unique to patients with respect to shape, size, and/or duration of eruptions. Nevertheless, the mechanism underlying multifarious eruptions in urticaria is largely unknown in medicine. The eruptions are believed to be evoked by histamine release from mast cells in the skin. However, the majority of visible characteristics of urticaria cannot be explained by a simple injection of histamine to the skin. In this study, we succeeded in developing a mathematical model that

can explain various geometrical shapes of eruptions typically observed in patients. Our mathematical model suggests that simultaneous self-regulation of positive and negative feedback of histamine through mast cells plays a critical role in generating the wide-spread wheal patterns. The study findings increase the understanding of the pathogenesis of urticaria and may aid decision-making for appropriate treatments. It also opens an entirely new avenue for mathematical approaches to analyze various skin diseases with geometric eruptions and predict the effectiveness of treatments through in silico experiments.

Computation of Fold and Cusp Bifurcation points in a System of Ordinary Differential Equations using Lagrange Multiplier Method



Prof. Matheus Tuwankotta

The Bandung Institute of Technology

We study fold abd cusp bifurcations in system of ordinary differential equations depending on one or two parameters. In particular, we prove that the fold bifurcation point in that system corresponds to a local extremum of a constrained optimization problem, which can be computed using the classical Lagrange Multiplier Method. Conversely, we provide a sufficient condition for a particular solution of a constrained optimization problem be in correspondence with a fold bifurcation of the two ordinary differential equations. We derive a similar result for cusp bifurcation in a system with two parameters. These results are applied to three examples to highlight the advantage of this proposed method.

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Normalizing field flow:solving forward and inverse stochastic differential equations using physics-informed flow model



Prof. Tao Zhou Chinese Academy of Sciences

We introduce normalizing field flows (NFF) for learning random fields from scattered measurements. More precisely, we construct a bijective transformation (a normalizing flow characterizing by neural networks) between a reference random field (say, a Gaussian random field with the Karhunen-Lo\`eve (KL) expansion structure) and the target stochastic field, where the KL expansion coefficients and the invertible networks are trained by maximizing the sum of the log-likelihood on scattered measurements. This NFF model can be used to solve data-driven forward, inverse, and mixed forward/inverse stochastic partial differential equations in a unified framework. We demonstrate the capability of the proposed NFF model for learning Non-Gaussian processes, mixed Gaussian processes, and forward & inverse stochastic partial differential equations.